The three gap theorem

(1950's Steinhaus conjecture)

For any $\alpha \in \mathbb{R}$, and any $N \in \mathbb{N} = \{1, 2, \dots\}$

there are at <u>most</u> three gaps.

<u>Proof</u> we call a gap <u>special</u> if the gap (x_i, x_{i+1}) doesn't map in to another gap (x_j, x_{j+1})

<u>Notice</u>, every gap has the same type as some special gap.

Prove this observation Exercise

How many special gaps are there?

We can get special gaps in two ways:

① There is a gap (x_i, x_{i+1}) but

$R^{-1}([0])$ is not there (+1)

(2) If an endpoint of (x_i, x_{i+1})

is the last rotation $R^N([0])$

(+2) two new types

$\square$

Note: the largest gap has length equal to the sum of that of the two smaller gaps.